

# Speed Control of Dc Motor Based On Linear Quadratic Regulator Method

Ishak ERTUGRUL<sup>1</sup>, Osman ULKIR<sup>2</sup>, Nihat AKKUS<sup>3</sup>

<sup>1,2,3</sup>Department of Mechatronics Engineering, Marmara University, Istanbul, 34722, Turkey

## Abstract

The aim of this paper is to propose a method for controlling the speed of a DC motor by using Linear quadratic regulator (LQR) and PID controllers. LQR ensures an optimal control law for a linear system. LQR algorithm is one of the most sophisticated methods to control a system. In this study, the LQR and PID controller methods were simulated with MATLAB to control the dc motor. The main goal of this controller is to reduce the errors caused due to the deviation of the speed of dc motor. The performance of the designed LQR and classic PID speed controller is compared and analyzed. Consequently, the result shows that the LQR approach has minimum overshoot, minimum transient and steady state parameters, which indicates that LQR is more effective and efficient than classic PID controller.

**Keywords:** Linear Quadratic Regulator (LQR), Optimal Control, PID Controller, DC Motor, Speed Control.

## 1. Introduction

DC motor is designed to convert electrical power into mechanical power. In DC motor, electrical energy is translated into mechanical energy through the interaction of two magnetic fields. DC motors are greatly used in industrial applications, robot systems and home appliances where speed control of motor is necessary. DC motors are most appropriate for great range speed control and are hereby used in many alterable speed drives due to their high reliabilities, flexibilities and low costs [1, 2].

Different approaches have been reported in literatures for controlling of the DC Motor. Genetic Algorithm, neural network, fuzzy based approach, particle swarm optimization techniques and linear quadratic regulator are the most important among these many works [3, 4].

Ertunc and Sezen [5] presented a simulation of the speed control of a DC motor using Fuzzy Logic Control (FLC) at Matlab/Simulink environment. The speed response of a DC motor exposed to fixed armature voltage was examined for both under loaded and unloaded operating conditions. Similarly, many studies [5-7] have used FLC for dc motor control.

Thomas and Poongodi [8] introduced position control of DC Motor using genetic algorithm based PID controller. Authors presented design a position controller of a DC motor by selection of a PID parameters using genetic algorithm.

The other type of control methods can be advanced such as Linear- Quadratic Regulator (LQR) optimal control, linear quadratic regulator design technique is well known in modern optimal control theory and has been greatly used a large number of studies. It has a very good robustness feature [4]. Some authors like [9-10] used a optimal strategies for speed control of dc motor through the linear quadratic regulator (LQR) and PID controller.

This paper presents speed control of dc motor by using of Linear Quadratic Regulator (LQR) technique and classic PID controller method. The results of LQR method compared with traditional PID controller method. Additionally, although this motor characteristics used in other studies, the experimental results obtained in this study provide better results than other studies. The rest of the paper is presented at first the dc motor mathematical and electrical model is described. The next section describes and designs the PID and LQR technique. Then experiment results are represented. In conclusion, the last section includes paper conclusion.

## 2. Dc Motor Model

A basic DC motor model is shown in Fig.1. The armature circuit consist of a resistance (R), related in series with an inductance (L), and a voltage source ( $\mathcal{E}_b$ ) representing the back electromotive (back emf) force induced in the armature when during rotation. [11-13].

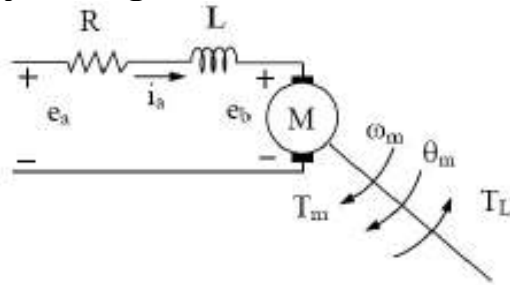


Figure 1: Dc motor model [23]

The motor torque  $T_m$  is connected to the armature current,  $i_a$ , by a torque constant  $k$ . In addition, said that;

$$\begin{aligned} k_t &= k_b = k \\ T_m &= k_t i_a \end{aligned} \quad (1)$$

The back emf,  $e_b$ , is relative to angular velocity by;

$$e_b = k_b \omega_m = k_b \frac{d\theta}{dt} \quad (2)$$

From the Figure 1, the following equations depend on Newton's Law and Kirchoff's Law has been had.

$$L \frac{di_a}{dt} + R i_a = e_a - k_b \frac{d\theta}{dt} \quad (3)$$

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = k_t i_a \quad (4)$$

State-space form explained that the equations above can be expressed by choosing the rotating speed and electrical current as the states variables and the voltage as an input. The output is chosen to be the rotating speed [1, 14].

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (5)$$

According to eq. from (1) to (5), the state space model will be:

$$\begin{bmatrix} \dot{i}_a \\ \dot{\omega}_m \\ \dot{\theta}_m \end{bmatrix} = \begin{bmatrix} -R/L & -k/L & 0 \\ k/J & -B/J & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix} e_a \quad (6)$$

$$\omega_m = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} \quad (7)$$

All experiments were performed according to the following parameters. These parameters are has been shown in the Table 1. The DC motor parameters are taken for this work [Wadhwani and Kushwah]

Table 1: Parameters of dc motor [14]

Parameter	Symbol	Value
Armature voltage	V	200V
Armature inductance	L	0.02H
Armature resistance	R	0.5Ω
Friction coefficient	B	0.008 N.m/r/s
Back emf const.	k	1.25 V/r/s
Motor torque const.	k	0.5 N.m/A
Mechanical inertia	J	0.1 Kg.m <sup>2</sup>

### 3. Linear Quadratic Regulator Method

Linear quadratic regulator technique is well known in modern optimal control theory and has been properly used in many academic and industrial study. The liner quadratic regulator method research to provide the optimal controller that reduces a given cost function. This cost function is formed by two matrices, Q and R, that weight the state vector and the system input respectively. [15].

To meet the appropriate statement, the controller is designed using LQR methodology. The cost function be described;

$$J = \int_0^{\infty} [x^T Q x + u^T R u] dt \quad (8)$$

Where Q is weighting factors of positive semi definite matrix and R is weighting factors of positive definite matrix. According to design the LQR controller, the first step is to choose the weighting matrices Q and R. The value R weight inputs more than the states while the value of Q weight the state more than the inputs. Then the feedback K can be calculated [1,15];

$$u = -Kx \quad (9)$$

K is the constant feedback gain obtained from the solution of the discrete algebraic Riccati equation. Where, the gain matrix K which analyses the LQR design [1].

$$K = R^{-1}B^T P \quad (10)$$

Positive semi definite solution to the Riccati equation for P[1,15];

$$A^T P + P.A - PBR^{-1}B^T P + Q = 0 \quad (11)$$

#### 4. Experimental Results

The results of the system with using PID and optimal LQR controller of experiment are shown this section. The responses of the dc motor system with PID and LQR controller design are being applied.

Fig. 2 demonstrates the step response of PID control. The PID controlled response of the system has significantly higher overshoot and big settling time values.

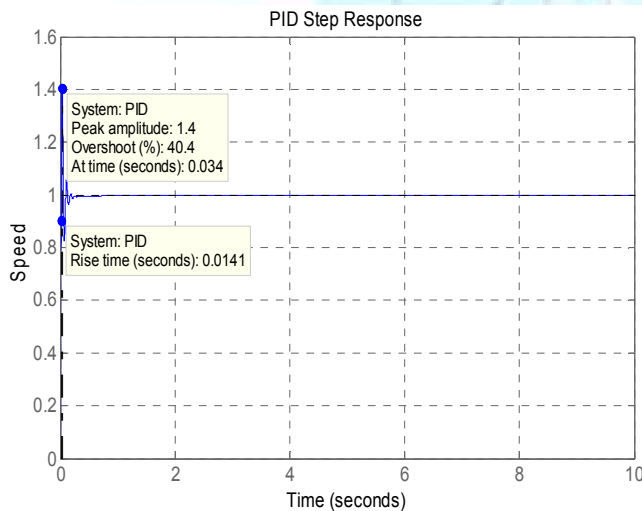


Figure 2: PID step response

Fig. 3 shows the step response of Linear Quadratic Regulator controller. LQR method controlled response of the system has greatly least settling time and smaller overshoot rates.

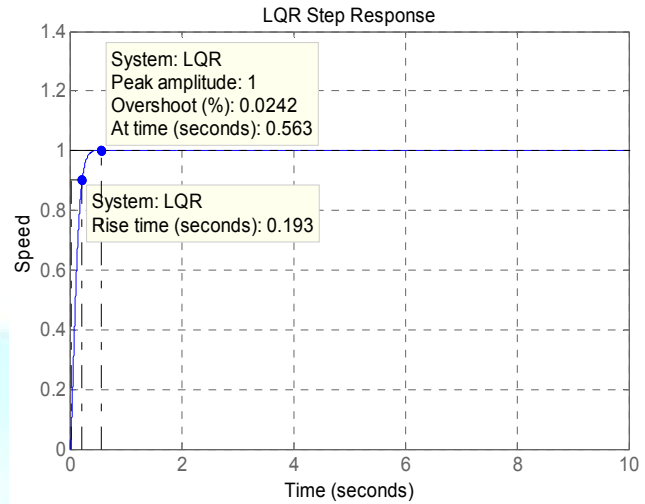


Figure 3: LQR step response

Fig. 4 shows comparison step response for PID controller and LQR controller system.

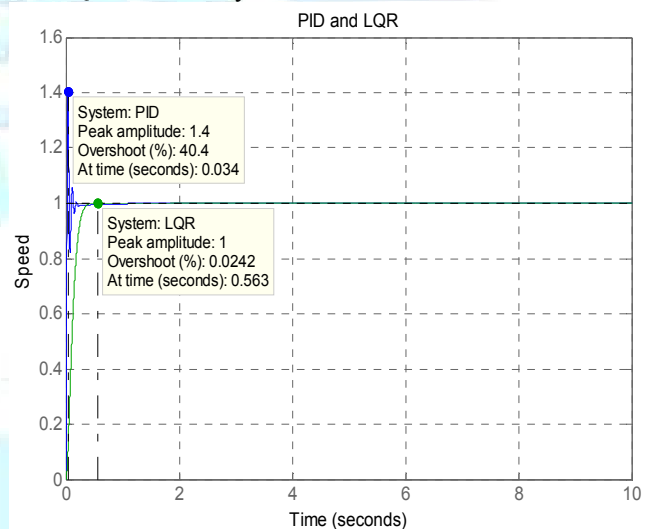


Figure 4: PID and LQR step response

Table 2: Comparison of results of PID and optimal LQR controller

	PID	LQR
PEAK TIME	0.034	0.563
RISE TIME	0.0141	0.193
OVERSHOOT(%)	40	0.0242
SETTLING TIME	0.162	0.322

#### 5. Conclusions

Speed control of a DC motor is an important topic, so this paper offers a design method to determine the speed control using LQR method and PID controller. The

simulation results are acquired using MATLAB program. The performance of the two controllers is confirmed through simulations. Experiments results are offered for comparison. Depend on the comparative simulation results, one can decide that the linear quadratic regulator controller achieves a robust dynamic performance of the DC motor with peak time, settling time, rise time and overshoot compared to classical PID controller under formal condition. The experiment results demonstrate that the peak time, overshoot, settling time and rise time has been advanced substantially by using Linear Quadratic Regulator (LQR) controller. The proposed Linear Quadratic Regulator controller has more advantages, such as higher flexibility, control, better dynamic and static performance compared with conventional PID controller. Therefore, LQR controller design was proposed and implemented.

## References

- [1] Isa, Mohd, and Nur Iznice Afrah. DC Motor Controller Using Linear Quadratic Regulator (LQR) Algorithm Implementation on PIC. Diss. Universiti Malaysia Pahang, 2008.
- [2] Mickky, Anand, and Pratibha Tiwari. "Analysis of Speed Control of Separately Excited DC Motor Using FOPID with LQR." *Analysis* 3.3 (2015).
- [3] Kanojiya, R.G. ve Meshram, P.M., "Optimal Tuning of PI Controller for Speed Control of DC motor drive using Particle Swarm Optimization", *Advances in Power Conversion and Energy Technologies (APCET)*, International Conference, s:1-6, 2012
- [4] Tripathi, Nikhita, et al. "Optimization Speed Control of DC Separately Excited Motor Using Tuning Controller of Linear Quadratic Regulator (LQR) Technique." *International Journal of Engineering Trends and Technology (IJETT)* – Volume 9 Number 10 - Mar 2014
- [5] Tipsuwan, Yodyium, and Mo-Yuen Chow. "Fuzzy logic microcontroller implementation for DC motor speed control." *Industrial Electronics Society, 1999. IECON'99 Proceedings. The 25th Annual Conference of the IEEE. Vol. 3. IEEE, 1999.*
- [6] Thepsatarn, P., et al. "DC motor speed control using fuzzy logic based on LabVIEW." *SICE-ICASE, 2006. International Joint Conference. IEEE, 2006.*
- [7] Teeter, Jason T., Mo Yuen Chow, and James J. Brickley Jr. "A novel fuzzy friction compensation approach to improve the performance of a DC motor control system." *Industrial Electronics, IEEE Transactions on* 43.1 (1996): 113-120.
- [8] Thomas, Neenu, and Dr P. Poongodi. "Position control of DC motor using genetic algorithm based PID controller." *Proceedings of the World Congress on Engineering. Vol. 2. 2009.*
- [9] Yu, Gwo-Ruey, and Rey-Chue Hwang. "Optimal PID speed control of brush less DC motors using LQR approach." *Systems, Man and Cybernetics, 2004 IEEE International Conference on. Vol. 1. IEEE, 2004.*
- [10] Gupta, Narendra K. "Frequency-shaped cost functionals-Extension of linear-quadratic-Gaussian design methods." *Journal of Guidance, Control, and dynamics* 3.6 (1980): 529-535.
- [11] Katsuhiko Ogata: *Modern Control Engineering*; Prentice Hall International, Inc. Fourth Edition 2002
- [12] Ogata, Katsuhiko. *System dynamics. Vol. 3.* New Jersey: Prentice Hall, 1998.
- [13] Hummadi, Ruba MK Al-Mulla. "Simulation Of Optimal Speed Control For A DC Motor Using Linear Quadratic Regulator (LQR)." *Journal* 3 (2012).
- [14] Wadhvani, Sulochana, Veena Verma, and Rekha Kushwah. "Design and Tuning of PID Controller Parameters based on Fuzzy Logic and Genetic Aalgorithm." *Int. Conf. On soft computing*, (2013).
- [15] Dwivedi, Dohare. "PID Conventional Controller and LQR Optimal controller for Speed analysis of DC Motor: A Comparative Study", *International Research Journal of Engineering and Technology (IRJET)*, Volume: 02 Issue: 08 | Nov-2015